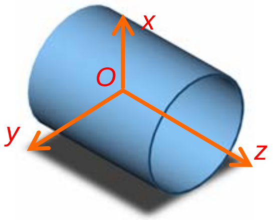
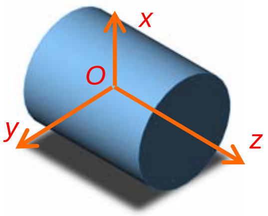
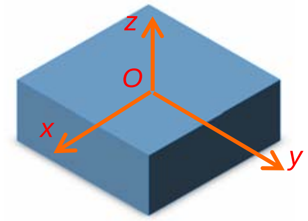
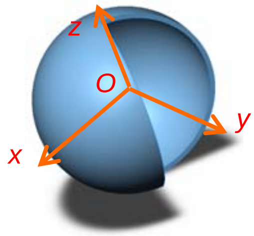
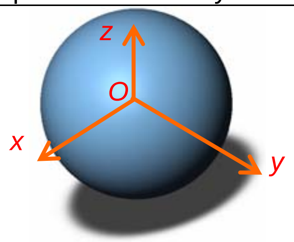
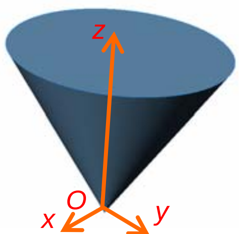
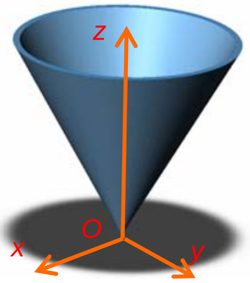
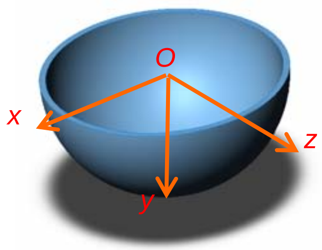
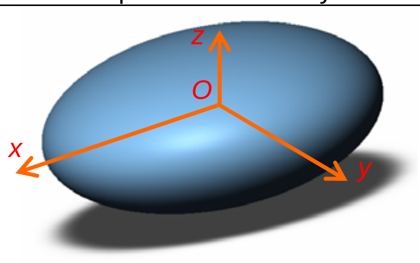
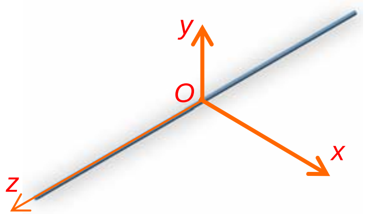
Géométrie des masses de solides homogènes

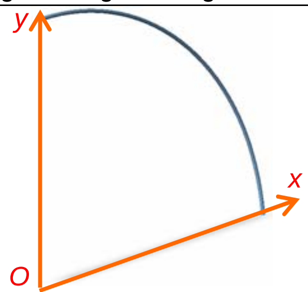
Corps homogène de masse m	Centre d'inertie	Matrice d'inertie en $(O,\vec{x},\vec{y},\vec{z})$
 cylindre creux : rayon R et longueur l	centre	$\begin{pmatrix} \frac{1}{2}mR^2+\frac{1}{12}ml^2 & 0 & 0 \\ 0 & \frac{1}{2}mR^2+\frac{1}{12}ml^2 & 0 \\ 0 & 0 & mR^2 \end{pmatrix}$
 cylindre plein : rayon R et longueur l	centre	$\begin{pmatrix} \frac{1}{4}mR^2+\frac{1}{12}ml^2 & 0 & 0 \\ 0 & \frac{1}{4}mR^2+\frac{1}{12}ml^2 & 0 \\ 0 & 0 & \frac{1}{2}mR^2 \end{pmatrix}$
 parallélépipède rectangle : coté a, b, c	centre	$\begin{pmatrix} \frac{1}{12}m(b^2+c^2) & 0 & 0 \\ 0 & \frac{1}{12}m(a^2+c^2) & 0 \\ 0 & 0 & \frac{1}{12}m(a^2+b^2) \end{pmatrix}$
 sphère creuse : rayon R	centre	$\begin{pmatrix} \frac{2}{3}mR^2 & 0 & 0 \\ 0 & \frac{2}{3}mR^2 & 0 \\ 0 & 0 & \frac{2}{3}mR^2 \end{pmatrix}$
 sphère pleine : rayon R	centre	$\begin{pmatrix} \frac{2}{5}mR^2 & 0 & 0 \\ 0 & \frac{2}{5}mR^2 & 0 \\ 0 & 0 & \frac{2}{5}mR^2 \end{pmatrix}$
 cône plein : rayon R, hauteur h	$z_C=\frac{3h}{4}$	$\begin{pmatrix} \frac{3m}{5}\left(\frac{R^2}{4}+h^2\right) & 0 & 0 \\ 0 & \frac{3m}{5}\left(\frac{R^2}{4}+h^2\right) & 0 \\ 0 & 0 & \frac{3m}{5}\frac{R^2}{2} \end{pmatrix}$

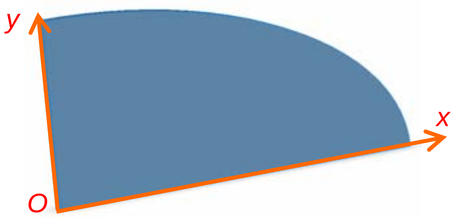
<table>
<tr><th>Corps homogène de masse m</th><th>Centre d'inertie</th><th>Matrice d'inertie</th></tr>
<tr><td>

cône creux : rayon R, hauteur h</td><td>$z_C = \frac{2h}{3}$</td><td>$$\begin{pmatrix} \frac{1}{4}mR^2+\frac{1}{2}mh^2 & 0 & 0 \\ 0 & \frac{1}{4}mR^2+\frac{1}{2}mh^2 & 0 \\ 0 & 0 & \frac{1}{2}mR^2 \end{pmatrix}$$</td></tr>
<tr><td>

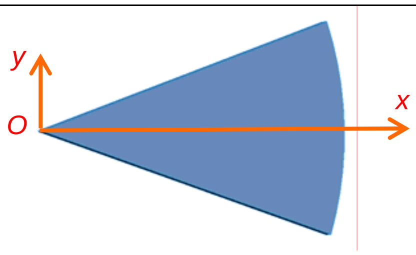
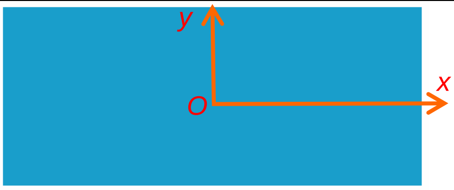
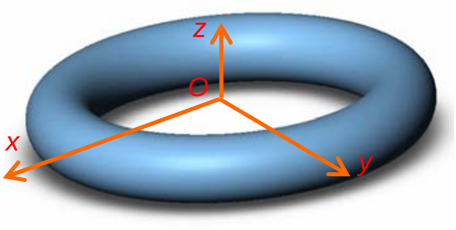
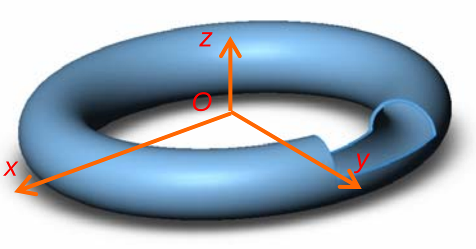
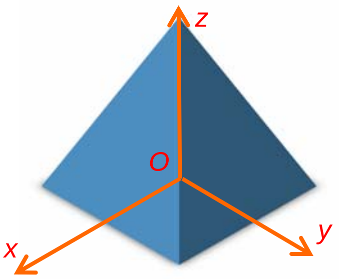
demi sphère creuse : rayon R</td><td>$z_C = \frac{R}{2}$</td><td>$$\begin{pmatrix} \frac{2}{3}mR^2 & 0 & 0 \\ 0 & \frac{2}{3}mR^2 & 0 \\ 0 & 0 & \frac{2}{3}mR^2 \end{pmatrix}$$</td></tr>
<tr><td>

ellipsoïde : axes $2a$, $2b$, $2c$</td><td>centre</td><td>$$\begin{pmatrix} \frac{1}{3}m\left(b^2+c^2\right) & 0 & 0 \\ 0 & \frac{1}{3}m\left(a^2+c^2\right) & 0 \\ 0 & 0 & \frac{1}{3}m\left(a^2+b^2\right) \end{pmatrix}$$</td></tr>
<tr><td>

tige rectiligne : longueur $2a$</td><td>centre</td><td>$$\begin{pmatrix} \frac{1}{3}ma^2 & 0 & 0 \\ 0 & \frac{1}{3}ma^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$</td></tr>
<tr><td>

quart de cercle : rayon R</td><td>$x_C = y_C = \frac{2R}{\pi}$</td><td>$$\begin{pmatrix} \frac{1}{2}mR^2 & \frac{1}{\pi}mR^2 & 0 \\ \frac{1}{\pi}mR^2 & \frac{1}{2}mR^2 & 0 \\ 0 & 0 & mR^2 \end{pmatrix}$$</td></tr>
<tr><td>

quart de plaque elliptique : demi-axes a, b</td><td>$x_C = \frac{4a}{3\pi}$
$y_C = \frac{4b}{3\pi}$</td><td>$$\begin{pmatrix} \frac{1}{4}mb^2 & \frac{1}{2\pi}mab & 0 \\ \frac{1}{2\pi}mab & \frac{1}{4}ma^2 & 0 \\ 0 & 0 & \frac{1}{4}m(a+b)^2 \end{pmatrix}$$</td></tr>
</table>

Corps homogène de masse m	Centre d'inertie	Matrice d'inertie
 secteur circulaire : rayon R	$x_C = \frac{2}{3}R\frac{\sin\alpha}{\alpha}$	$\begin{pmatrix} \frac{1}{4}mR^2\left(1-\frac{\sin 2\alpha}{2\alpha}\right) & 0 & 0 \\ 0 & \frac{1}{4}mR^2\left(1+\frac{\sin 2\alpha}{2\alpha}\right) & 0 \\ 0 & 0 & \frac{1}{2}mR^2 \end{pmatrix}$
 rectangle : a et b	centre	$\begin{pmatrix} \frac{4}{3}mb^2 & 0 & 0 \\ 0 & \frac{4}{3}ma^2 & 0 \\ 0 & 0 & \frac{4}{3}m\left(a^2+b^2\right) \end{pmatrix}$
 tore plein : rayons R et a	centre	$\begin{pmatrix} m\left(\frac{a^2}{2}+\frac{5R^2}{8}\right) & 0 & 0 \\ 0 & m\left(\frac{a^2}{2}+\frac{5R^2}{8}\right) & 0 \\ 0 & 0 & m\left(a^2+\frac{3R^2}{4}\right) \end{pmatrix}$
 tore creux : rayons R et a	centre	$\begin{pmatrix} m\left(\frac{a^2}{2}+\frac{5R^2}{4}\right) & 0 & 0 \\ 0 & m\left(\frac{a^2}{2}+\frac{5R^2}{4}\right) & 0 \\ 0 & 0 & m\left(a^2+\frac{3R^2}{2}\right) \end{pmatrix}$
 pyramide : a, b, h	$x_C = \frac{h}{4}$	$\begin{pmatrix} m\left(\frac{b^2}{20}+\frac{h^2}{10}\right) & 0 & 0 \\ 0 & m\left(\frac{a^2}{20}+\frac{h^2}{10}\right) & 0 \\ 0 & 0 & \frac{m}{20}\left(a^2+b^2\right) \end{pmatrix}$