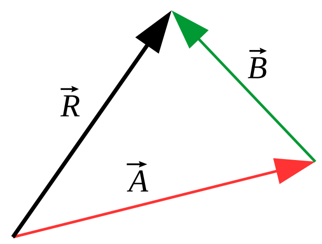
Trigonométrie - Projections

Pourquoi la trigonométrie est si importante en ingénierie ?

Utilisation de modèle mathématiques :

Ingénierie électrique ⇒ tension (v) et courant (A) modélisés par des vecteurs

Ingénierie mécanique ⇒ force (N), couple (N.m), distance (m), vitesse de translation (m/s) vitesse de rotation (rad/s), accélération linéaire (m/s²), accélération angulaire (rad/s²) ⇒ modélisés par des vecteurs



$$\vec{R}=\vec{A}+\vec{B}$$

$$\|\vec{A}\|=4{,}1$$

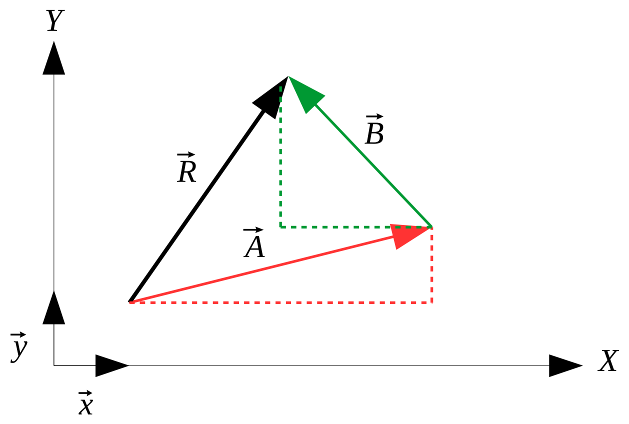
$$\|\vec{B}\|=2{,}83$$

En quoi est utile la trigonométrie avec les vecteurs ?

Les vecteurs (forces, tension, etc) se somment vectoriellement !
Dand le cas présent $\|\vec{R}\|=3{,}9$

Il n'est pas possible d'ajouter des vecteurs algébriquement. $\|\vec{A}\|+\|\vec{B}\|=4{,}1+2{,}83=6{,}93 \neq 3{,}9$

⇒ solution : sommer algébriquement les composantes.



<u>Trigonométrie</u>

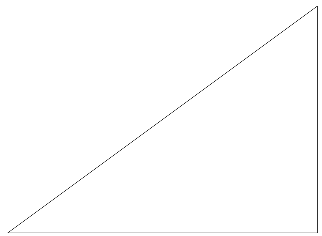
<u>Sinus – Cosinus - Tangente</u> $\xrightarrow{\textit{Mnémotechnique}}$ Soh Cah Toa

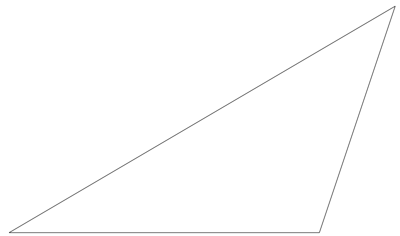
Soh Cah Toa

Tan

Cos

Sin





Domaine de validité :

Trigonométrie

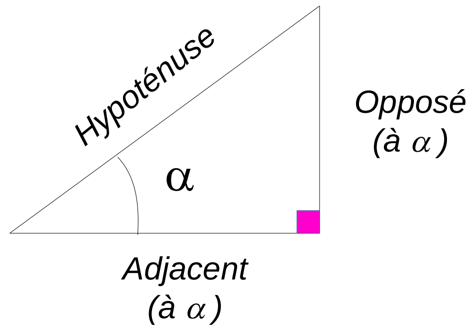
<u>Sinus – Cosinus - Tangente</u> $\xrightarrow{\textit{Mnémotechnique}}$ Soh Cah Toa

Soh Cah Toa

$$\tan\alpha = \frac{Opp}{Adj}$$

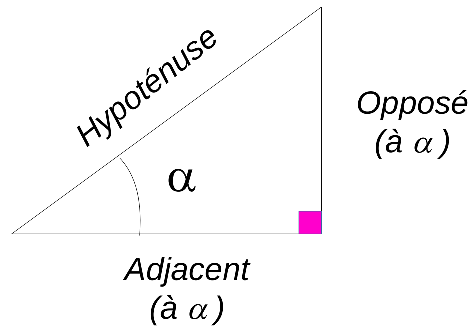
$$\cos\alpha = \frac{Adj}{Hyp}$$

$$\tan\alpha = \frac{Opp}{Hyp}$$



$$Adj = Hyp \cdot \cos\alpha$$

$$Opp = Hyp \cdot \sin\alpha$$



Trigonométrie

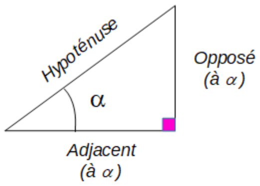
Sinus – Cosinus - Tangente $\xrightarrow{\text{Mnémotechnique}}$ Soh Cah Toa

Soh Cah Toa

Toa: $\tan\alpha = \dfrac{Opp}{Adj}$

Cah: $\cos\alpha = \dfrac{Adj}{Hyp}$

Soh: $\tan\alpha = \dfrac{Opp}{Hyp}$



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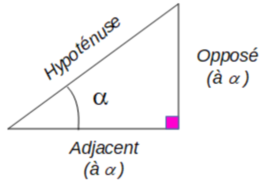
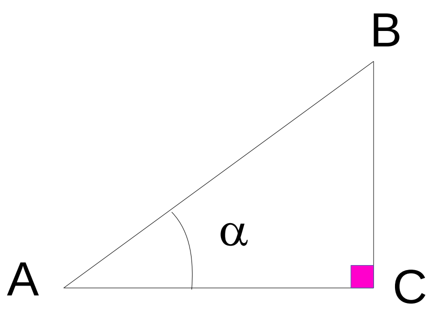
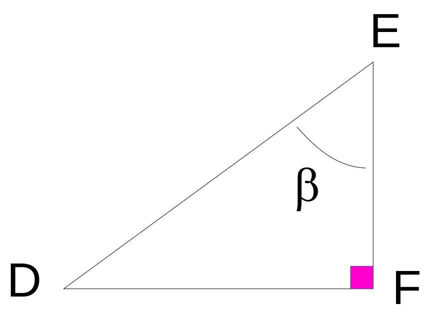


Application 1



Application 2



Trigonométrie

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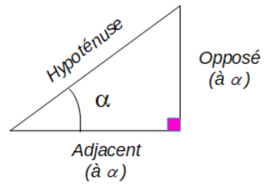
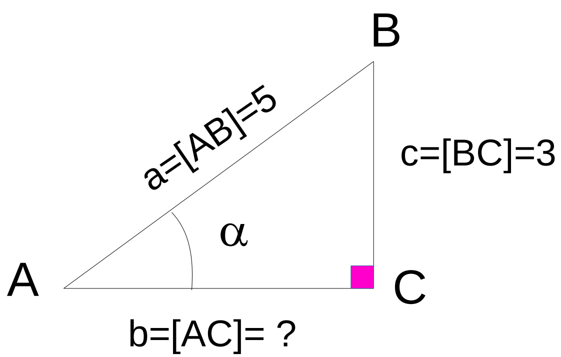


Application 3



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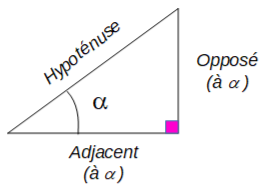
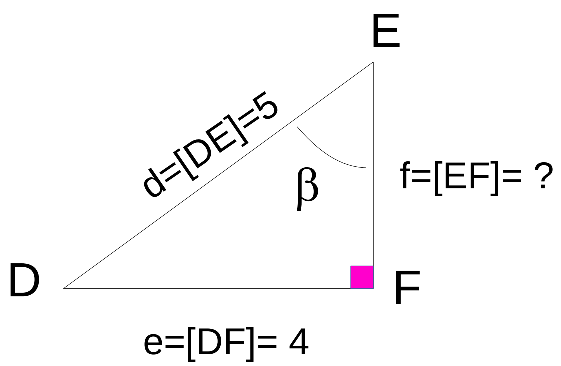
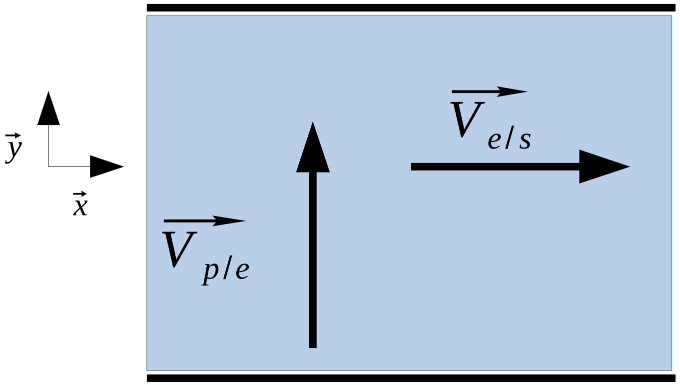


Règle des maçons

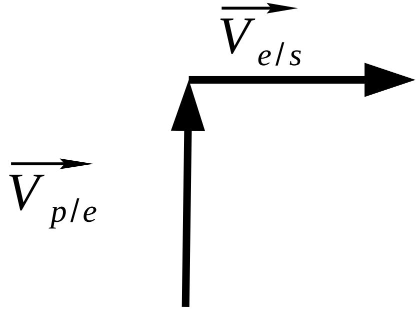
Application 4



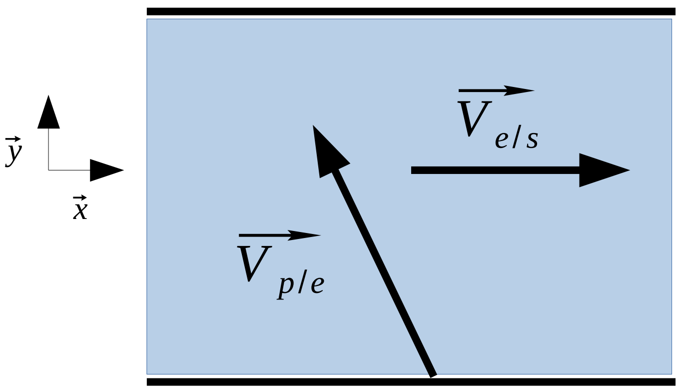
Projections



$$\overrightarrow{V}_{p/s} = \overrightarrow{V}_{p/e} + \overrightarrow{V}_{e/s}$$



$$\|\overrightarrow{V}_{p/e}\| = \|\overrightarrow{V}_{e/s}\| = 3\, m/s$$



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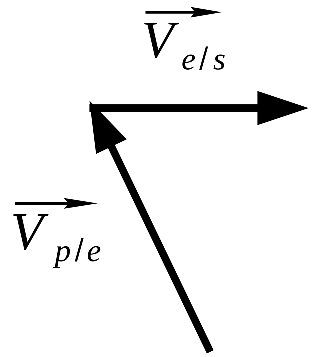
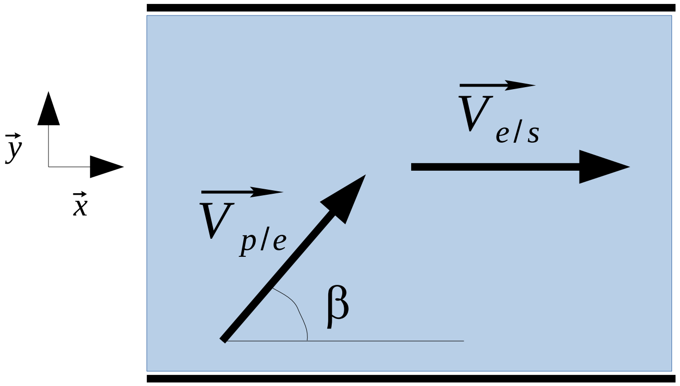


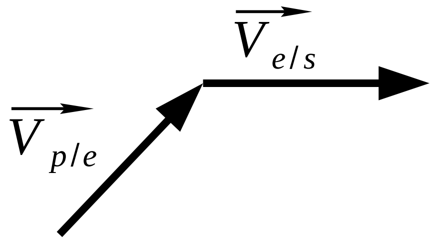
$$\|\overrightarrow{V}_{p/e}\| = \|\overrightarrow{V}_{e/s}\| = 3\, m/s$$

$$\overrightarrow{V}_{p/e} = -2 \cdot \vec{x} + 3 \cdot \vec{y}$$

Projections



$$\overrightarrow{V_{p/s}} = \overrightarrow{V_{p/e}} + \overrightarrow{V_{e/s}}$$



$$\|\overrightarrow{V_{p/e}}\| = \|\overrightarrow{V_{e/s}}\| = 3\,m/s$$

$$\beta = 30°$$

Projections

<u>Objectif possible :</u> réaliser un bilan des actions mécaniques extérieures afin de déterminer l'accélération de la pièce 1 de masse m.

$$\sum \overrightarrow{F_{ext}} = \frac{d\,\vec{p}}{dt} = d\,\frac{(m\ \vec{v})}{dt} = m\ \cdot d\,\frac{\vec{v}}{dt} = m\ \cdot \vec{a} \qquad \text{avec} \qquad \vec{P} = m\ \cdot \vec{v}$$

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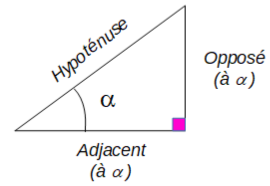

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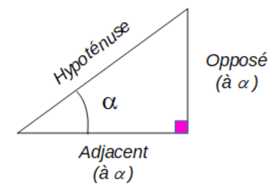
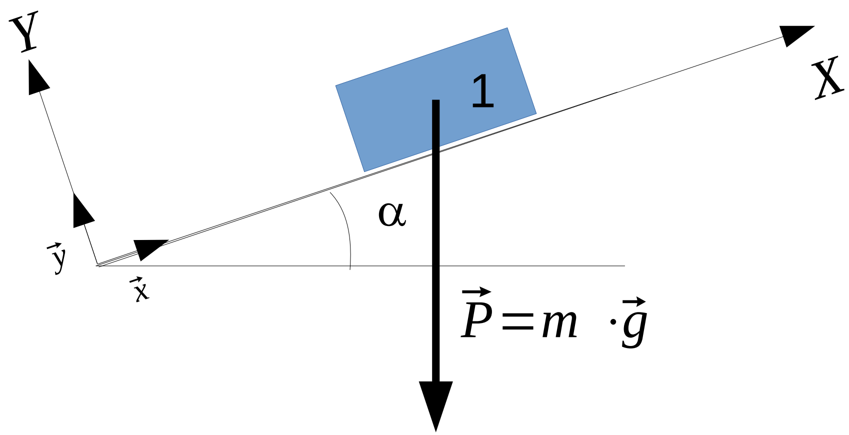
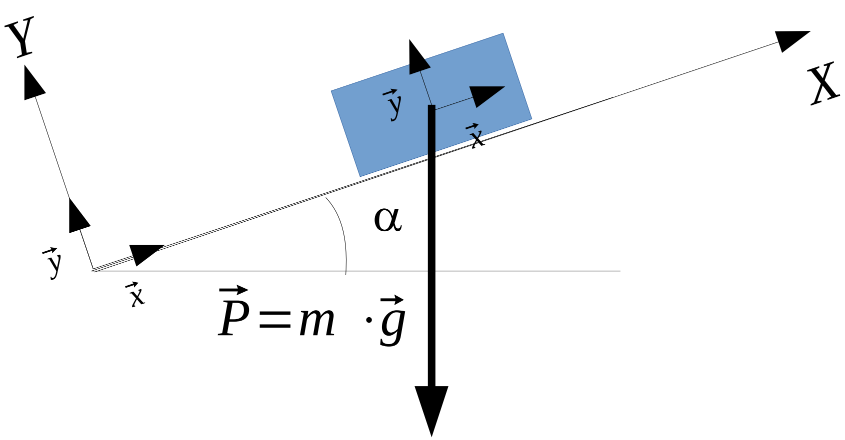
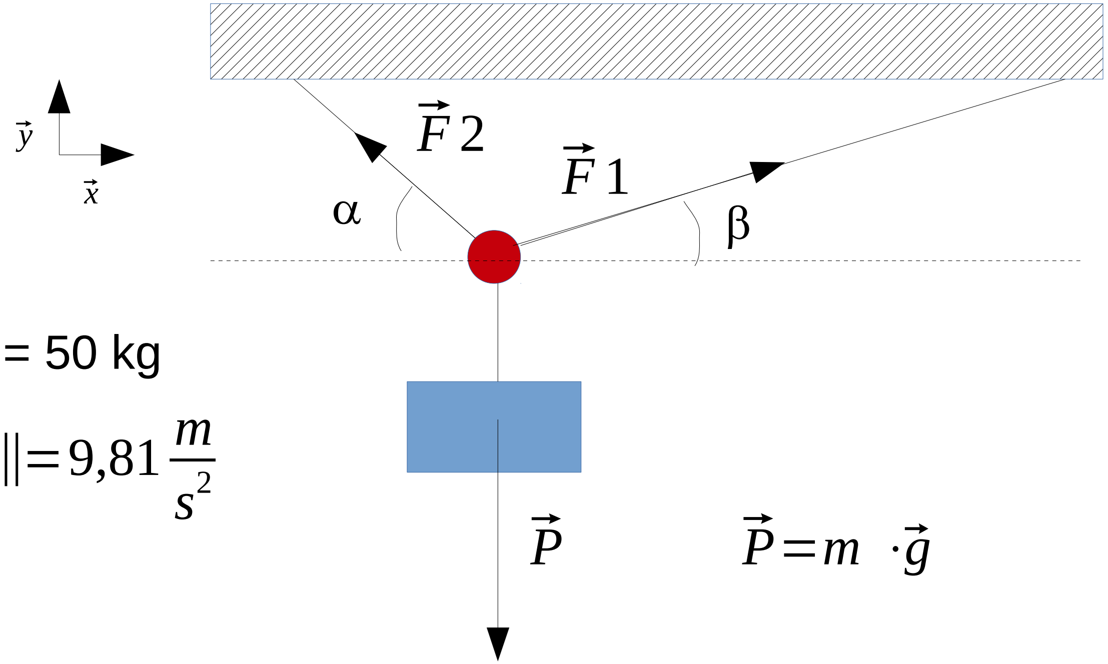






Projections

Objectif possible : réaliser un bilan des actions mécaniques extérieures afin de déterminer les efforts F1 et F2



m = 50 kg

$$||\vec{g}|| = 9{,}81 \frac{m}{s^2}$$

$$\vec{P} = m \cdot \vec{g}$$

Sinus – Cosinus - Tangente $\xrightarrow{\text{Mnémotechnique}}$ Soh Cah Toa

Soh Cah Toa

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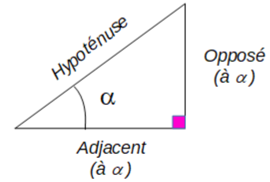

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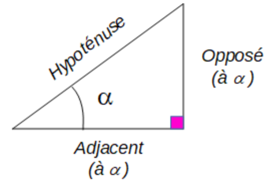

Projections

Cas particulier des vecteurs unitaires

Projections utilisées dans les figures planes en cinématique

L'objectif pourrait être le suivant :

Exprimer la vitesse suivante $V_{a\in 2/1}=4\cdot\vec{x}_1+2\cdot\vec{y}_1$ dans le repère $(O_0,\vec{x}_{0,}\vec{y}_0)$